

FERMION MASSES IN SUSY GUTS¹

B. C. Allanach² and S. F. King³

*Physics Department, University of Southampton
Southampton, SO17 1BJ*

Abstract

We discuss the fermion mass problem in SUSY GUTs, including such ideas as texture zeroes, and Georgi-Jarlskog textures. We focus on a specific supersymmetric model based on the gauge group $SU(4) \otimes SU(2)_L \otimes SU(2)_R$. In this model the gauge group is broken to that of the standard model at 10^{16} GeV, and supersymmetry is broken at low-energy. The model may be regarded as a “surrogate SUSY GUT”, and has several advantages over the $SU(5)$ and $SO(10)$ SUSY GUTs, such as the absence of the doublet-triplet splitting problem, and a simpler Higgs sector which does not involve any large Higgs representations. The model predicts full quadruple Yukawa unification (top-bottom-tau-tau neutrino), leading to the prediction of large top mass and $\tan \beta$. An operator analysis leads to schemes where the strange and down mass are predicted, and $|V_{ub}| > 0.004$.

¹Invited talk at the International Workshop on Elementary Particle Physics: Present and Future, Valencia 1995.

²Address after 1 October 1995: Rutherford Appleton Laboratory, Chilton, Didcot, OX11 0QX.

³Talk presented by S. F. King.

1 Introduction

The Standard Model involves three gauge couplings g_i , ($i = 1, 2, 3$), a Higgs mass parameter μ and quartic coupling λ , and the fermion Yukawa couplings. In the Standard Model Lagrangian one must specify the three Yukawa matrices λ_{ij}^E , λ_{ij}^D , λ_{ij}^U , corresponding to up to 54 real parameters, which after diagonalisation lead to 9 physical masses (6 quark masses and 3 charged lepton masses) and 4 physical quark mixing parameters. Thus the fermion sector of the Standard Model involves either 54 or 13 unknown free parameters, depending on how you choose to count them. Either way, from a fundamental point of view the situation is unacceptable and the fermion mass problem, as it has been called, is one motivation for going beyond the Standard Model. The big question of course is what lies beyond it?

We have not yet experimentally studied the mechanism of electroweak symmetry breaking, so one might argue that it is premature to study the fermion mass problem. Unless we can answer this, we have no hope of understanding anything about fermion masses since we do not have a starting point from which to analyse the problem. However LEP has taught us that whatever breaks electroweak symmetry must do so in a way which very closely resembles the standard model. This observation by itself is enough to disfavour many dynamical models involving large numbers of new fermions. By contrast the minimal supersymmetric standard model (MSSM) mimics the standard model very closely. Furthermore, by accurately measuring the strong coupling constant, LEP has shown that the gauge couplings of the MSSM merge very accurately at a scale just above 10^{16} GeV, thus providing a hint for possible unification at this scale. On the theoretical side, supersymmetry (SUSY) and grand unified theories (GUTs) fit together very nicely in several ways, providing a solution to the technical hierarchy problem for example. When SUSY GUTs are extended to supergravity (SUGRA) the beautiful picture of universal soft SUSY breaking pa-

rameters and radiative electroweak symmetry breaking via a large top quark yukawa coupling emerges. Finally, there is an on-going effort to embed all of this structure in superstring models, thereby allowing a complete unification, including gravity.

Given the promising scenario mentioned above, it is hardly surprising that many authors have turned to the SUSY GUT framework as a springboard from which to attack the problem of fermion masses [1]. Indeed in recent years there has been a flood of papers on fermion masses in SUSY GUTs. Although the approaches differ in detail, there are some common successful themes which have been known for some time. For example the idea of bottom-tau Yukawa unification in SUSY GUTs [2] works well with current data [3]. A more ambitious extension of this idea is the Georgi-Jarlskog (GJ) ansatz (see later) which provides a successful description of all down-type quark and charged lepton masses [4], and which also works well with current data [5]. In order to understand the origin of the texture zeroes, one must consider the details of the model at the scale $M_X \sim 10^{16}$ GeV. The alternative is to simply make a list of assumptions about the nature of the Yukawa matrices at M_X [6]. For example Ramond, Roberts and Ross (RRR) [7] assumed symmetric Yukawa matrices at M_X , together with the GJ ansatz for the lepton sector. It is difficult to proceed beyond this without specifying a particular model. Indeed, this model dependence may be a good thing since it may mean that the fermion mass spectrum at low energies is sensitive to the theory at M_X , so it can be used as a window into the high-energy theory. Therefore in what follows we shall restrict ourselves to a very specific gauge group at M_X .

Twenty-one years ago Pati and Salam proposed a model in which the standard model was embedded in the gauge group $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ [8]. More recently a superpersymmetric (SUSY) version of this model was proposed in which the gauge group is broken at $M_X \sim 10^{16}$ GeV [9]. The model [9] does not involve adjoint representations and later some attempt was made to derive it from four-dimensional

strings, although there are some difficulties with the current formulation [10]. The absence of adjoint representations is not an essential prerequisite for the model to descend from the superstring, but it leads to some technical simplifications. Also in the present model, the colour triplets which are in separate representations from the Higgs doublets, become heavy in a very simple way so the Higgs doublet-triplet splitting problem does not arise. These two features (absence of adjoint representations and absence of the doublet-triplet splitting problem,) are shared by flipped $SU(5)\otimes U(1)$ [11], which also has a superstring formulation. Although the present model and flipped $SU(5)\otimes U(1)$ are similar in many ways, there are some important differences. Whereas the Yukawa matrices of flipped $SU(5)\otimes U(1)$ are completely unrelated at the level of the effective field theory at M_X (although they may have relations coming from the string model) in the present model there is a constraint that the top, bottom and tau Yukawa couplings must all unify at that scale. In addition there will be Clebsch relations between the other elements of the Yukawa matrices, assuming they are described by non-renormalisable operators, which would not be present in flipped $SU(5)\otimes U(1)$. In these respects the model resembles the $SO(10)$ model recently analysed by Anderson et al [12]. However it differs from the $SO(10)$ model in that the present model does not have an $SU(5)$ subgroup which is central to the analysis of the $SO(10)$ model. In addition the operator structure of the present model is totally different. Thus the model under consideration is in some sense similar to flipped $SU(5)\otimes U(1)$, but has third family Yukawa unification and precise Clebsch relationships as in $SO(10)$. We find this combination of features quite remarkable, and it seems to us that this provides a rather strong motivation to study the problem of fermion masses in this model [13].

2 The 422 Model

Here we briefly summarise the parts of the model which are relevant for our analysis.

The gauge group is,

$$\text{SU}(4) \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R. \quad (1)$$

The left-handed quarks and leptons are accommodated in the following representations,

$$F^{i\alpha a} = (4, 2, 1) = \begin{pmatrix} u^R & u^B & u^G & \nu \\ d^R & d^B & d^G & e^- \end{pmatrix}^i \quad (2)$$

$$\bar{F}_{x\alpha}^i = (\bar{4}, 1, \bar{2}) = \begin{pmatrix} \bar{d}^R & \bar{d}^B & \bar{d}^G & e^+ \\ \bar{u}^R & \bar{u}^B & \bar{u}^G & \bar{\nu} \end{pmatrix}^i \quad (3)$$

where $\alpha = 1 \dots 4$ is an $\text{SU}(4)$ index, $a, x = 1, 2$ are $\text{SU}(2)_{L,R}$ indices, and $i = 1 \dots 3$ is a family index. The Higgs fields are contained in the following representations,

$$h_a^x = (1, \bar{2}, 2) = \begin{pmatrix} h_2^+ & h_1^0 \\ h_2^0 & h_1^- \end{pmatrix} \quad (4)$$

(where h_1 and h_2 are the low energy Higgs superfields associated with the MSSM.)

The two heavy Higgs representations are

$$H^{ab} = (4, 1, 2) = \begin{pmatrix} u_H^R & u_H^B & u_H^G & \nu_H \\ d_H^R & d_H^B & d_H^G & e_H^- \end{pmatrix} \quad (5)$$

and

$$\bar{H}_{\alpha x} = (\bar{4}, 1, \bar{2}) = \begin{pmatrix} \bar{d}_H^R & \bar{d}_H^B & \bar{d}_H^G & e_H^+ \\ \bar{u}_H^R & \bar{u}_H^B & \bar{u}_H^G & \bar{\nu}_H \end{pmatrix}. \quad (6)$$

The Higgs fields are assumed to develop VEVs,

$$\langle H \rangle = \langle \nu_H \rangle \sim M_X, \quad \langle \bar{H} \rangle = \langle \bar{\nu}_H \rangle \sim M_X \quad (7)$$

leading to the symmetry breaking at M_X

$$\text{SU}(4) \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R \longrightarrow \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \quad (8)$$

in the usual notation. Under the symmetry breaking in Eq.8, the Higgs field h in Eq.4 splits into two Higgs doublets h_1, h_2 whose neutral components subsequently

develop weak scale VEVs,

$$\langle h_1^0 \rangle = v_1, \quad \langle h_2^0 \rangle = v_2 \quad (9)$$

with $\tan \beta \equiv v_2/v_1$.

Below M_X the part of the superpotential involving quark and charged lepton fields is just

$$W = \lambda_U^{ij} Q_i \bar{U}_j h_2 + \lambda_D^{ij} Q_i \bar{D}_j h_1 + \lambda_E^{ij} L_i \bar{E}_j h_1 + \dots \quad (10)$$

with the boundary conditions at M_X ,

$$\lambda_U^{ij} = \lambda_D^{ij} = \lambda_E^{ij}. \quad (11)$$

The same Yukawa relations also occur in minimal $SO(10)$.

3 The Basic Strategy

Such Yukawa relations as in Eq.11 taken at face value are a phenomenological disaster. For example consider the minimal $SU(5)$ prediction $\lambda_D^{ij} = \lambda_E^{ij}$. After diagonalisation this leads to $\lambda_e = \lambda_d$, $\lambda_\mu = \lambda_s$, $\lambda_\tau = \lambda_b$, (at the scale M_X) and hence

$$\frac{\lambda_s}{\lambda_d} = \frac{\lambda_\mu}{\lambda_e} \quad (12)$$

which is RG invariant and fails badly at low-energy. On the other hand the third family relation leads to the low-energy prediction (assuming the SUSY RG equations) $\lambda_b/\lambda_\tau \approx 2.4$ which works well.

A possible fix is provided by the GJ texture,

$$\lambda^E = \begin{pmatrix} 0 & \lambda_{12} & 0 \\ \lambda_{21} & 3\lambda_{22} & 0 \\ 0 & 0 & \lambda_{33} \end{pmatrix} \quad (13)$$

$$\lambda^D = \begin{pmatrix} 0 & \lambda_{12} & 0 \\ \lambda_{21} & \lambda_{22} & 0 \\ 0 & 0 & \lambda_{33} \end{pmatrix}, \quad (14)$$

With predictions (at M_X)

$$\lambda_s = \lambda_\mu/3, \quad \lambda_d = 3\lambda_e, \quad (15)$$

which is viable.

As it turns out, the idea of full top-bottom-tau Yukawa unification works rather well for the third family [14], leading to the prediction of a large top quark mass $m_t > 165$ GeV, and $\tan\beta \sim m_t/m_b$ where m_b is the bottom quark mass. However Yukawa unification for the first two families is not successful, since it would lead to unacceptable mass relations amongst the lighter fermions, and zero mixing angles at M_X . In order to cure these problems, we require something akin to the GJ texture, in which the Yukawa relations are altered by group theoretical Clebsch coefficients, leading to enhanced predictivity.

One interesting proposal has recently been put forward to account for the fermion masses in an SO(10) SUSY GUT with a single Higgs in the 10 representation [12]. According this approach, only the third family is allowed to receive mass from the renormalisable operators in the superpotential. The remaining masses and mixings are generated from a minimal set of just three specially chosen non-renormalisable operators whose coefficients are suppressed by some large scale. Furthermore these operators are only allowed to contain adjoint 45 Higgs representations, chosen From a set of fields denoted 45_Y , 45_{B-L} , $45_{T_{3R}}$, 45_X whose VEVs point in the direction of the generators specified by the subscripts, in the notation of [12]. This is precisely the strategy we wish to follow. We shall assume that only the third family receives its mass from a renormalisable Yukawa coupling. All the other renormalisable Yukawa couplings are set to zero. Then non-renormalisable operators are written down which will play the role of small effective Yukawa couplings. The effective Yukawa couplings are small because they originate from non-renormalisable operators which are suppressed by powers of the heavy scale M . We shall restrict ourselves to all possible

non-renormalisable operators which can be constructed from different group theoretical contractions of the fields:

$$O_{ij} \sim (F_i \bar{F}_j) h \left(\frac{H \bar{H}}{M^2} \right)^n + \text{h.c.} \quad (16)$$

where we have used the fields H, \bar{H} in Eqs.5,6 and M is the large scale $M > M_X$. The idea is that when H, \bar{H} develop their VEVs such operators will become effective Yukawa couplings of the form $h F \bar{F}$ with a small coefficient of order M_X^2/M^2 . Although we assume no intermediate symmetry breaking scale (i.e. $\text{SU}(4) \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R$ is broken directly to the standard model at the scale M_X) we shall allow the possibility that there are different higher scales M which are relevant in determining the operators. For example one particular contraction of the indices of the fields may be associated with one scale M , and a different contraction may be associated with a different scale M' . We shall either appeal to this kind of idea in order to account for the various hierarchies present in the Yukawa matrices, or to higher dimensional operators which are suppressed by a further factor of M .

In the present model, although there are no adjoint representations, there will in general be non-renormalisable operators which closely resemble those in $\text{SO}(10)$ involving adjoint fields. The simplest such operators correspond to $n = 1$ in Eq.16, with the $(H \bar{H})$ group indices contracted together.

These operators are similar to those of ref. [12] but with $H \bar{H}$ playing the rôle of the adjoint Higgs representations. It is useful to define the following combinations of fields, corresponding to the different $H \bar{H}$ transformation properties under the gauge group in Eq.1,

$$\begin{aligned} (H \bar{H})_A &= (1, 1, 1) \\ (H \bar{H})_B &= (1, 1, 3) \\ (H \bar{H})_C &= (15, 1, 1) \\ (H \bar{H})_D &= (15, 1, 3) \end{aligned} \quad (17)$$

It is straightforward to construct the operators of the form of Eq.16 explicitly, and hence deduce the effect of each operator. For example for $n = 1$ the four operators are, respectively,

$$O_{ij}^{A,B,C,D} \sim F_i \bar{F}_j h \frac{(H \bar{H})_{A,B,C,D}}{M^2} + H.c. \quad (18)$$

where we have suppressed gauge group indices.

These operators lead to quark-lepton and isospin splittings, as shown explicitly below:

$$\begin{aligned} O_{ij}^A &= a_{ij}(Q_i \bar{U}_j h_2 + Q_i \bar{D}_j h_1 + L_i \bar{E}_j h_1 + H.c.) \\ O_{ij}^B &= b_{ij}(Q_i \bar{U}_j h_2 - Q_i \bar{D}_j h_1 - L_i \bar{E}_j h_1 + H.c.) \\ O_{ij}^C &= c_{ij}(Q_i \bar{U}_j h_2 + Q_i \bar{D}_j h_1 - 3L_i \bar{E}_j h_1 + H.c.) \\ O_{ij}^D &= d_{ij}(Q_i \bar{U}_j h_2 - Q_i \bar{D}_j h_1 + 3L_i \bar{E}_j h_1 + H.c.) \end{aligned} \quad (19)$$

where the coefficients of the operators $a_{ij}, b_{ij}, c_{ij}, d_{ij}$ are all of order $\frac{M_X^2}{M^2}$.

4 Results and Conclusions

Using operators such as those above, together with more complicated $n = 2$ operators, it is possible to account for the entire fermion mass spectrum. The successful ansatz [13] involve 8 real parameters plus an unremovable phase. With these 8 parameters we can describe the 13 physical masses and mixing angles. Third family Yukawa unification leads to a prediction for $m_t(\text{pole}) = 130 - 200$ GeV and $\tan \beta = 35 - 65$, depending on $\alpha_S(M_Z)$ and \bar{m}_b . More accurate predictions could be obtained if the error on $\alpha_S(M_Z)$ and \bar{m}_b were reduced. The analysis of the lower 2 by 2 block of the Yukawa matrices leads to 2 possible predictions for $\lambda_\mu/\lambda_s = 3, 4$ at the scale M_X (3 is the GJ prediction). In the upper 2 by 2 block analysis we are led to 5 possible predictions for $\lambda_d/\lambda_e = 2, 8/3, 3, 4, 16/3$. (again 3 is the GJ prediction.) Finally, we have a prediction that $|V_{ub}| > 0.004$ [13].

The high values of $\tan\beta$ required by our model (also predicted in SO(10)) can be arranged by a suitable choice of soft SUSY breaking parameters as discussed in ref.[15], although this leads to a moderate fine tuning problem [14]. The high value of $\tan\beta$ is not stable under radiative corrections unless some other mechanism such as extra approximate symmetries are invoked. m_t may have been overestimated, since for high $\tan\beta$, the equations for the running of the Yukawa couplings in the MSSM can get corrections of a significant size from Higgsino–stop and gluino–sbottom loops. The size of this effect depends upon the mass spectrum and may be as much as 30 GeV. For our results to be quantitatively correct, the sparticle corrections to m_b must be small. This could happen in a scenario with non-universal soft parameters, for example. Not included in our analysis are threshold effects, at low or high energies. These could alter our results by several per cent and so it should be borne in mind that all of the mass predictions have a significant uncertainty in them. It is also unclear how reliable 3 loop perturbative QCD at 1 GeV is.

Despite a slight lack of predictivity of the model compared to SO(10), the $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ model has the twin advantages of having no doublet-triplet splitting problem, and containing no adjoint representations, making the model technically simpler to embed into a realistic string theory. Although both these problems can be addressed in the SO(10) model [16], we find it encouraging that such problems do not arise in the first place in the $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ model. Of course there are other models which also share these advantages such as flipped SU(5) or even the standard model. However, at the field theory level, such models do not lead to Yukawa unification, or have precise Clebsch relations between the operators describing the light fermion masses. It is the combination of all of the attractive features mentioned above which singles out the present model for serious consideration.

References

- [1] S. Raby, Preprint No. OHSTPY-HEP-T-95-024; S. Raby, Preprint No. OHSTPY-HEP-T-95-001.
- [2] M. B. Einhorn and D. R. T. Jones, Nucl. Phys. **B196** (1982) 475; J. Ellis, D. V. Nanopoulos and S. Rudaz, Nucl. Phys. **B202** (1982) 43.
- [3] B. C. Allanach and S. F. King, Phys. Lett. **B328**, (1994) 360; H. Arason *et al*, Phys. Rev. Lett. **67** (1991) 2933; *ibid* Phys. Rev. **D47** (1993) 232; P. Langacker and N. Polonsky, UPR-0556T. V.Barger, M.S.Berger and P.Ohmann, Phys. Rev. **D47**, 1093 (1993).
- [4] H. Georgi and C. Jarlskog, Phys. Lett. **B86** (1979) 297.
- [5] S. Dimopoulos, L. Hall and S. Raby, Phys. Rev. **D45** (1992) 4192.
- [6] C. D. Froggatt and H. B. Nielson, Nucl. Phys. **B147** (1979) 277; Z. G. Berezhiani, Phys. Lett. **B129** (1983) 99; *ibid.*, **B150** (1985) 177; S. Dimopoulos, Phys. Lett. **B129** (1983) 417.
- [7] P. Ramond, R. Roberts and G. Ross, Nucl. Phys. **B406** (1993) 19.
- [8] J. Pati and A. Salam, Phys. Rev. **D10** (1974) 275.
- [9] I. Antoniadis and G. K. Leontaris, Phys. Lett. **B216** (1989) 333.
- [10] I. Antoniadis, G. K. Leontaris and J. Rizos, Phys. Lett. **B245** (1990) 161.
- [11] I. Antoniadis, J. Ellis, J. Hagelin and D. Nanopoulos, Phys. Lett. **B194** (1987) 231.
- [12] G. Anderson, S. Raby, S. Dimopoulos, L. Hall and G. Starkman, Phys. Rev **D49** (1994) 3660

- [13] S. F. King Phys. Lett. **B325** (1994) 129;
 B. C. Allanach and S. F. King, hep-ph/9502219, submitted to Nucl. Phys. **B**.
- [14] B. Ananthanarayan, G. Lazarides and Q. Shafi, Phys. Rev. **D44** (1991) 1613;
 H. Arason *et al*, Phys. Rev. Lett. **67** (1991) 2933; L. Hall, R. Rattazzi and U.
 Sarid, LBL preprint 33997.
- [15] M. Carena, M. Olechowski, S. Pokorski and C. E. M. Wagner, Nucl. Phys. **B426**
 (1994) 269.
- [16] S. Chaudhuri, S. Chung and J. Lykken, FERMILAB PUB-94/137-T; L. Hall and
 S. Raby, OHSTPY-HEP-T-94-023, LBL-36357, UCB-PTH-94/27.